Stochastic Analysis, 2025 Spring: HW1

February 26, 2025

From Le Gall: Exercises 1.15, 1.16, 1.18, 2.25.

Additional exercises:

Exercise 1 (Transformation of BM) Recall that BM is a continuous centered Gaussian process with covariance

$$\mathsf{E} B_t B_s = s \wedge t. \tag{1}$$

Show that all the following processes are centered Gaussian with the same covariance (1).

1. $(-B_t)_{t\geq 0}$.
2. $(B_t^{\lambda})_{t\geq 0} \coloneqq (\frac{1}{\lambda} B_{\lambda^2 t})_{t\geq 0}$. (Fix $\lambda > 0$.)
3. $(B_t^{(s)})_{t\geq 0} \coloneqq (B_{t+s} - B_s)_{t\geq 0}$. (Fix $s > 0$.)